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Absolute measurement of activity of a volumetric object by collimated detectors: Solid angle issues

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Abstract

A Monte Carlo method has been developed to determine that part of a radioactive volumetric object, which is common to the fields of view of collimated detectors combined for the absolute measurement of the activity of the object. Furthermore, the method evaluates the solid angle subtended by the detectors at this part of the object and allows an optimised positioning of the detectors in order to achieve a maximum possible solid angle. Photon self-absorption within the object is demonstrated to be an important parameter which, depending on the photon energy, geometry and the material of the object, may affect the size of the volume of intersection and hence the solid angle.

Keywords: Monte Carlo; Radioactivity; Solid angle; Collimation

1. Introduction

The solid angle Ω, subtended by a bare or collimated detector from a radioactive object, is an essential parameter in the absolute measurement of activity through the nuclear radiation emitted by the object. Both analytical (Gardner and Verghese, 1971) and Monte Carlo (Williams, 1966; Nicolaou et al., 1987) approaches have been employed to evaluate Ω for different geometries. Although the former is straightforward for the case of a point source, one has to opt for the Monte Carlo approach when complicated geometries with volumetric objects are involved.

In a variety of radiation physics situations, collimated detectors are combined, for example in coincidence or a sum mode, in the absolute measurement of activity of an object. In this case, only part of the object is effectively analysed. This part is a region in 3-dimensions (3D) within the object, formed from the intersection of the fields of view of the detectors and comprises points subtending a non-zero solid angle from them. The importance of this region, namely volume of intersection, is twofold. Firstly, exact knowledge of the part of the object effectively analysed, allows the accurate determination of the solid angle essential in the absolute measurement of activity. Secondly, it permits optimum, reliable and reproducible positioning in complicated experimental arrangements where several collimated detectors are involved.

The aim of this study is to develop a method in order to determine the volume of intersection, within a radioactive object, of collimated detectors and to evaluate the solid angle subtended by the detectors from this volume.

2. Evaluation of the solid angle subtended by the volume of intersection

The method is based on the fact that the volume of intersection within an object encloses points, which subtend a non-zero solid angle from all the detectors. The evaluation

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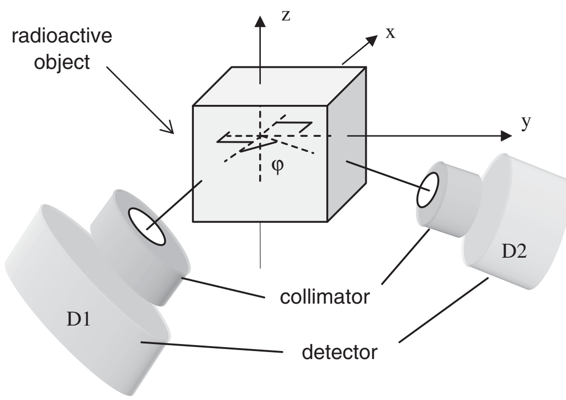

Fig. 1. The arrangement of two collimated detectors and a cubic radioactive object.

of the solid angle subtended by a detector from a point is performed using a Monte Carlo approach utilising total variance reduction (Wielopolski, 1977; Nicolaou et al., 1987).

Fig. 1 shows in 3D a cubic radioactive object and two collimated detectors D1 and D2 at an angle φ to each other. The central axes of the two detectors are on the (x, y) plane. The object is assumed to have a uniform activity distribution and to emit photon radiation isotropically. A computer program was written in FORTRAN to calculate the solid angle through the simulation of the activity distribution in the object, the photon emission directions and the geometry involved. A random number generator creates, firstly, points emitting photons isotropically and then the random directions of the emitted photons.

The (X, Y, Z) coordinates of a generated point within the object are given, along the x, y, z directions, as follows:

$$X = \mathrm{SL}(\mathrm{RN1} - 0.5),\quad Y = \mathrm{SL}(\mathrm{RN2} - 0.5),\quad Z = \mathrm{SL}(\mathrm{RN3} - 0.5), \tag{1}$$

where RN1, RN2 and RN3 are three independent random numbers equidistributed in [0, 1] and SL is the side of the object. Then, in relation to detector D1 the position of the generated point is

$$H = H0 + X,\quad P = \sqrt{(Y^2 + Z^2)}, \tag{2}$$

where H and P are the distances of the point from the face of D1 and the x-axis respectively, and $H0$ is the distance of the centre of the object from the face of D1.

In the case of the isotropic emission into a sphere of unit radius, the solid angle $\mathrm{d}\Omega$ in spherical coordinates is

$$\mathrm{d}\Omega = \sin\,\theta\,\mathrm{d}\theta\,\mathrm{d}\alpha, \tag{3}$$

where θ and α are the longitudinal and horizontal angles, respectively. The joint probability density distribution $P(\theta, \alpha)$ for the isotropic photon emission by the point, giving the fractional radiation emitted in $\mathrm{d}\Omega$, is

$$P(\theta, \alpha) \cdot \mathrm{d}\theta\,\mathrm{d}\alpha = \frac{\mathrm{d}\Omega}{4\pi} \tag{4}$$

yielding

$$P(\theta) = \sin\,\theta/2 \quad 0 \leqslant \theta \leqslant \pi, \tag{5}$$

$$P(\alpha) = \pi/2 \quad 0 \leqslant \alpha \leqslant 2\pi. \tag{6}$$

The random directions of photon emission are chosen through Eqs. (5) and (6) under the restriction that the sampled directions intersect the collimator aperture and the detector D1. Two weighting factors W_1 and W_2 are associated with the selection on θ and α. Then, the total weighting factor, for a given selection θ and α, is $W_i = W_1 W_2$ and represents the solid angle subtended by D1 from the particular selection. The procedure is repeated for a large number of random points generated within the object and the associated random directions of photon emission. The points with $W_i \neq 0$ fall in a 3D region within the object which constitutes the field of view of detector D1.

The volume of intersection of the fields of view of different detectors is the 3D region within the object comprising points, which subtend a non-zero solid angle from all the detectors. Then, only those points in the field of view of D1, which will also subtend a non-zero solid angle from D2, will define the volume of intersection.

Therefore, the next stage is to examine which of the points in the field of view of D1, subtend also a non-zero solid angle from detector D2. For this purpose, random directions are now generated according to Eqs. (5) and (6) for each of the points in the field of view of D1, under the restriction that the sampled directions intersect the collimator aperture and the detector D2. The total weighting factor for a given selection θ' and α' is then $W_i' = W_1' W_2'$ and represents the solid angle subtended by the collimated detector D2 from the particular selection. The points with $W_i' \neq 0$ fall in the 3D field of view of detector D2.

Then, the volume of intersection of the fields of view of detectors D1 and D2 comprises those points which satisfy the condition $W_i \neq 0$ and $W_i' \neq 0$. The solid angle subtended by detectors D1 and D2 from the volume of intersection is then given by

$$\Omega_{\mathrm{D1}} = \frac{1}{N} \sum_{i=1}^{N} W_i \tag{7}$$

and

$$\Omega_{\mathrm{D2}} = \frac{1}{M} \sum_{i=1}^{M} W_i', \tag{8}$$

where N and M are the number of random point positions, in relation to D1 and D2, respectively, satisfying the condition $W_i \neq 0$ and $W'_i \neq 0$.

3. Results and discussion

The concept of the region of intersection of the fields of view of detectors D1 and D2 and the associated solid angle are now demonstrated in 2-dimensions (2D). For this reason, an (x, y) cross-sectional plane at $z=0$ is considered through the two detectors and the object. This region will now be termed area of intersection.

The field of view, within the square object, of the collimated detector D1 is the dark region indicated in Fig. 2, under the assumption that the photons emitted by the random point positions are not attenuated inside the object. It contains points subtending a non-zero solid angle from the detector D1.

In order to obtain the area of intersection of the two detectors, the (x, y) coordinates of points at which D1 subtend a non-zero solid angle are transformed to (x', y') in the coordinate system of D2 through the matrix equation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{9}$$

which gives

$$x' = x \cdot \cos \varphi + y \cdot \sin \varphi,$$
$$y' = -x \cdot \sin \varphi + y \cdot \cos \varphi, \tag{10}$$

where the origins of the D1 and the D2 coordinate systems $[xy]$ and $[x'y']$, respectively, are the same but the x'-axis makes an angle φ with the positive x-axis. The area of intersection is then made of those points within the field of view of D1 for which D2 also subtends a non-zero solid angle (Fig. 3).

The variation of the solid angle that detector D2 subtends from the area of intersection is shown in Fig. 4, as a function of the rotation angle φ. In the region [110°, 116°] a plateau exists over which the solid angle does not vary significantly with the angle φ. An optimum positioning of the two detectors can be considered at $113° \pm 3°$. Over this range, the solid angle is effectively independent of φ. Hence, a reproducible detectors–object arrangement, with respect to the solid angle and area of intersection, can be achieved over a small range of values of φ. This is particularly important and advantageous for in situ analysis with bulky instrumentation of volumetric objects, where accurate reproducibility of the experimental geometry might be difficult to achieve.

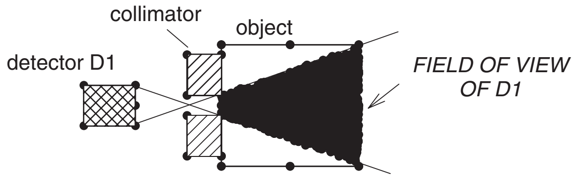

Fig. 2. A geometrical plot of the field of view of detector D1.

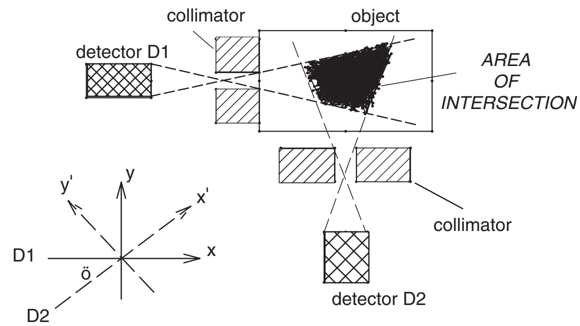

Fig. 3. A geometrical plot of the area of intersection between D1 and D2.

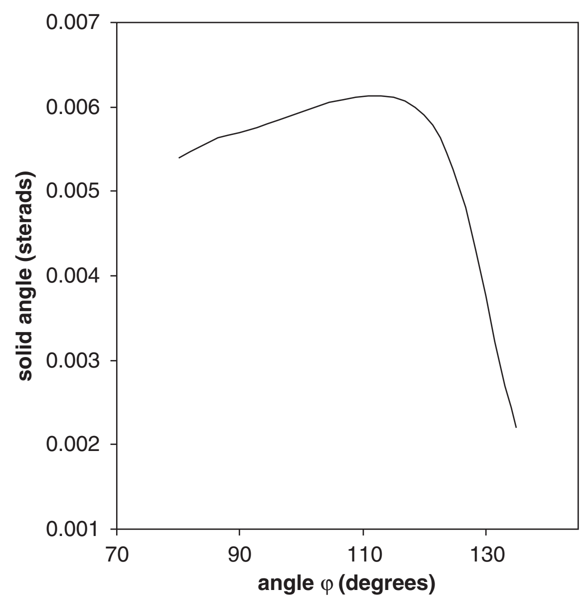

Fig. 4. Variation of the solid angle that detector D2 subtends at the area of intersection as a function of the rotation angle φ.

The effect of self-absorption within the object of the emitted photons was investigated. In this case, the number of photons reaching the detector is reduced by a factor $\exp(-\mu \cdot L)$, where μ is the linear attenuation coefficient of the material of the object at the photon energy of interest and L is the path length within the object of the particular simulated photon direction.

Fig. 5 shows the effect of self-absorption on the field of view of detector D1 for photon energy 100 keV and an object made of water. The light region in the field of view of

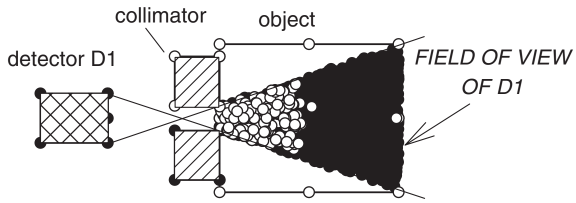

Fig. 5. The effect of self-absorption on the field of view of detector D1.

D1 emits photons, which are not attenuated in the object due to their small path length. The dark region, on the contrary, indicates the area from which the emitted photons are attenuated in the object due to their larger path length. Self-absorption has resulted in an effective field of view reduced compared to the one in Fig. 2 where it was assumed that photons were not attenuated inside the object. The solid angle of detector D1 at this reduced field of view is 0.0043 sterads, as opposed to 0.0067 sterads at the whole field of view in Fig. 2. Evidently, self-absorption may affect the field of view of a detector depending on the energy of the emitted photons, dimensions of the experimental arrangement and the material of the object. Consequently the solid angle Ω and the area of intersection of the two detectors would be affected, indicating that self-absorption should be considered in the measurement of activity.

4. Conclusions

A Monte Carlo method has been developed to determine that part of a radioactive volumetric object, which is common to the fields of view of collimated detectors combined for the absolute measurement of the activity of the object. Hence, the method determines the part, termed volume of intersection, of the object that is effectively analysed by the detectors. Furthermore, the method presents visually in 3D the volume of intersection and evaluates the solid angle subtended by each of the detectors from this part. The method was demonstrated in 2D for an arrangement of two collimated detectors at an angle $\varphi = 90°$ to each other and an object with isotropic photon emission. The variation of the solid angle for different values of φ indicated that, at the maximum solid angle, a plateau exists over which the variation is not significant.

The method presented can easily incorporate different geometrical configurations, objects of unusual shape, scattering inside the object (Nakamura, 1970; Horowitz et al., 1975), self-absorption and non-isotropic sources with an angular photon emission distribution. The importance of self-absorption, which may result in a reduced field of view, and hence solid angle and volume of intersection, was demonstrated. In the case of the 2D geometry studied, the solid angle subtended by a detector from the square object was reduced by 35% when self-absorption within the object was taken into consideration.

A computer program was written in FORTRAN to implement the method and calculate the solid angle. The execution time of the program depends on the number of histories generated for the points and photon directions, which in turn controls the error on the estimate of the solid angle (Wielopolski, 1977). For example, for 10 000 histories, the execution time on a commercial PII laptop with WINDOWS 98 as operating system was of the order of 15 s with an error of 1%.

The method is applicable in a variety of radiation physics situations, such as calculations on criticality, radiation dosimetry and shielding. The procedure determines, through evaluation of the solid angles involved, the intensity of a radiation beam at different points within a complicated 3D geometry. This intensity, in turn, determines dose levels and the necessary shielding and criticality issues when neutrons are involved.

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