Appl. Radiat. Isot. Vol. 37, No. 12, pp. 1219–1224, 1986
Int. J. Radiat. Appl. Instrum. Part A
Printed in Great Britain.

0883-2889/86 $3.00 + 0.00

The Effect of Solid Angle on the Reproducibility of an Experimental Set-up in Prompt Gamma-ray Neutron Activation Analysis

G. E. NICOLAOU,* Y. S. KHRBISH and N. M. SPYROU

Department of Physics, University of Surrey, Guildford, Surrey, GU2 5XH, U.K.

(*Received* 10 *April* 1986)

A Monte Carlo method which allows the calculation of the solid angle subtended by a collimated detector from surface and distributed sources is presented. The method utilizes total variance reduction and its validity is demonstrated for cylindrical and disc sources. The effect of rotation and displacement of the source on the solid angle is investigated for small and large sources. A measure of tolerance on the reproducibility of the experimental set-up in activity quantification is discussed.

Introduction

In order to establish an absolute or rather non-comparative method for the determination of multi-elemental concentrations using, for example, prompt gamma-ray neutron activation analysis (PGNAA) good and accurate knowledge of all the relevant nuclear parameters involved and reliable as well as reproducible experimental conditions are required. In particular, it is important to determine whether the size and orientation of the target have any effect on the absolute full energy peak efficiency of the detector.

The aim here is to relate any variation in the solid angle between source (target) and detector, as the target orientation is changed, to any variation in the geometrical efficiency. The solid angle is evaluated for a cylindrical target whose centre is located on the axis of a collimated detector, as shown in Fig. 1. The work is based upon a Monte Carlo method which utilizes total variance reduction (Wielopolski, 1975; Nicolaou, 1983).

In the PGNAA experiments a collimated beam of neutrons impinges upon a target and the prompt γ-rays produced in the capture process (n, γ) are collimated, by a straight bore hole collimator made of lead (20 mm dia.), to fall on a Ge(Li) detector. The semiconductor detector in our experimental set-up is at 200 mm from the target. The target can be oriented at any angle between 0° and 90° with the neutron beam direction.

Solid Angle for an Inclined Cylindrical Target

The computer program simulates the isotropic emission of photons uniformly distributed in the target. A random number generator is used to generate points of disintegration and the random directions of the emitted photons. Referring to Fig. 1, the variable position of the generated points is given as follows

$$Z = SL \cdot (X1 - 0.5)$$

$$R1 = RS \cdot \sqrt{X2} \quad \text{and} \quad \beta = 2 \cdot \pi \cdot X3$$

where $X1$, $X2$ and $X3$ are three independent random numbers equidistributed in [0, 1]. For an arbitrary angle of inclination (ϵ) of the target the variable position of the selected point is

$$P^2 = Z^2 + R1^2 - (Z \cdot \cos\epsilon + R1 \cdot \sin\epsilon \cdot \sin\beta)^2$$

$$H = H0 + Z \cdot \cos\epsilon + R1 \cdot \sin\epsilon \cdot \sin\beta$$

The program by utilising total variance reduction, forces the randomly selected directions originating from the target to go through the collimator aperture.

For isotropic emission into a unit sphere

$$P(\theta, \alpha) \cdot d\theta \cdot d\alpha = \frac{\Omega}{4\pi}$$

$$d\Omega = \sin\theta \cdot d\theta \cdot d\alpha$$

* Present address: Radiation and Nuclear Chemistry Division, Commission of the European Communities, Joint Research Centre, Ispra Establishment, 21020 Ispra, Varese, Italy.

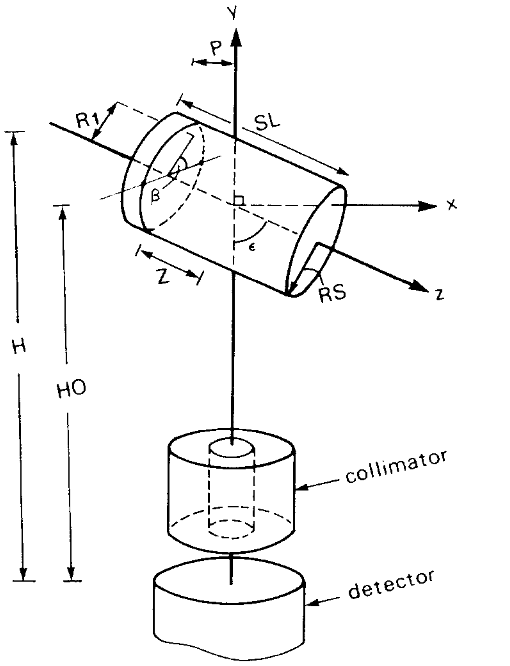

Fig. 1. Geometrical configuration of a collimated detector and a cylindrical target.

where θ, α are the longitudinal and horizontal angles respectively and $P(\theta, \alpha)$ is the joint probability density distribution for isotropic emission (Wielopolski, 1977). Then

$$P(\theta) = \frac{\sin\theta}{2} \quad 0 \leqslant \theta \leqslant \pi \tag{1}$$

$$P(\alpha) = \tfrac{1}{2}\pi \quad 0 \leqslant \alpha \leqslant 2\pi \tag{2}$$

The random directions are chosen by sampling equations (1) and (2). However, the free selection of θ and α has to be restricted so that the sampled direction intercepts the collimator aperture and the detector. Associated with the restrictions on θ and α are their respective weighting factors $W1$ and $W2$. The total weighting factor for any particular selection of θ and α is then $Wi = W1 \cdot W2$ where Wi represents the solid angle subtended by the detector for the particular selection. The solid angle $\Omega(P)$ for the configuration is

$$\Omega(P) = \frac{1}{N} \cdot \sum_{i=1}^{N} Wi \tag{3}$$

where N is the number of sampling histories.

In order to calculate the solid angle of the target subtended by a collimated detector, two cases are considered.

Case A

The point is located above the aperture at $P1$, Fig. 2 ($P < RC$). In this case θ_{max} remains constant and is equal to

$$\theta_{max} = \arctan\left(\frac{RC + P}{H - Q}\right)$$

There is also a critical angle θ_{CR1} below which α may vary over 2π and above which its variation is limited to $2\alpha_{max}$, where

$$\theta_{CR1} = \arctan\left(\frac{RC - P}{H - Q}\right)$$

At first a particular selection of θ will allow the determination of α_{max}. Defining X as a random number equidistributed in [0, 1], θ is given by

$$X = \frac{\int_0^{2\pi} \frac{\sin\theta}{2} \cdot d\theta}{\int_0^{\theta_{max}} \frac{\sin\theta}{2} \cdot d\theta}$$

$\therefore \theta = \arccos\,[1 - X \cdot (1 - \cos\theta_{max})]$

The weighting factor associated with this selection of θ is

$$W1 = \frac{\int_0^{\theta_{max}} \frac{\sin\theta}{2} \cdot d\theta}{\int_0^{\pi} \frac{\sin\theta}{2} \cdot d\theta}$$

and

$$W1 = \frac{1 - \cos\theta_{max}}{2}, \quad \text{for} \quad \theta_{max} \leqslant \theta_{C1}$$

$$W1 = \frac{1 - \cos\theta_{C1}}{2}, \quad \text{for} \quad \theta_{max} > \theta_{C1}$$

where $\theta_{C1} = \arctan\,[(RO + P)/H]$; if for a particular selection of θ, $\theta > \theta_{C1}$ then $W1 = 0$.

Once θ has been selected, the weighting factor associated with the selection of α is determined,

$$\text{if } \theta \leqslant \theta_{CR1}, \quad \alpha\text{: } [0, 2\pi] \quad \text{and} \quad W2 = 1$$

$$\text{if } \theta > \theta_{CR1}, \quad \alpha \leqslant 2\alpha_{max} \quad \text{and} \quad W2 = \frac{\alpha_{max}}{\pi}$$

The solid angle for this selection is then given by equation (3).

Case B

The point is located outside the aperture at $P2$, Fig. 3 ($P \geqslant RC$).

Given X as a random number equidistributed in [0, 1], the angle α is determined from

$$X = \frac{\int_{-\alpha_{max}}^{\alpha} \frac{d\alpha}{2\pi}}{\int_{-\alpha_{max}}^{\alpha_{max}} \frac{d\alpha}{2\pi}}$$

$$\therefore \alpha = \alpha_{max} \cdot (2X - 1), \quad -\alpha_{max} \leqslant \alpha \leqslant \alpha_{max}$$

where

$$\alpha_{max} = \arctan\left(\frac{RC}{\sqrt{(P^2 - RC^2)}}\right).$$

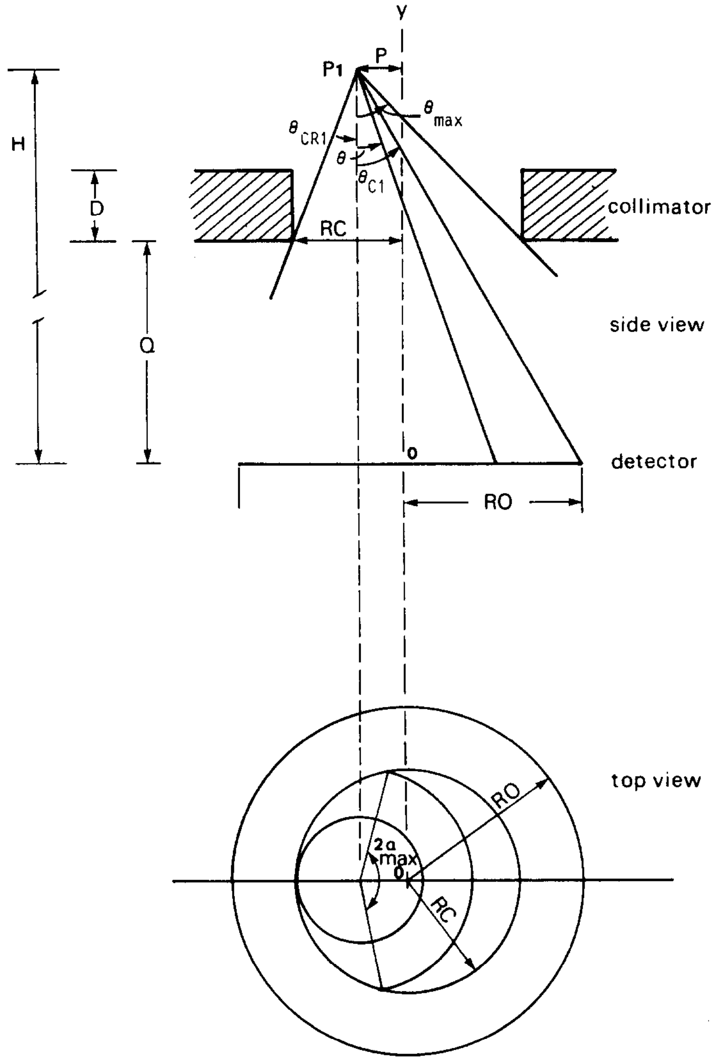

Fig. 2. Geometrical configuration of a point source above the aperture of the collimator.

The weighting factor $W2$ associated with this selection if α is

$$W2 = \frac{\int_{-\alpha_{max}}^{\alpha_{max}} \frac{d\alpha}{2\pi}}{\int_0^{2\pi} \frac{d\alpha}{2\pi}} \quad \therefore\ W2 = \frac{\alpha_{max}}{\pi}$$

Once α has been determined, θ_{min} and θ_{max} are obtained from Fig. 3, then

$$\theta_{min} = \arctan\left(\frac{P\cdot\cos\alpha - \sqrt{(RC^2 - P^2\cdot\sin^2\alpha)}}{H-(Q+D)}\right)$$

$$\theta_{max} = \arctan\left(\frac{P\cdot\cos\alpha + \sqrt{(RC^2 - P^2\cdot\sin^2\alpha)}}{H-Q}\right)$$

A critical angle θ_{CR2} exists which tests whether the randomly chosen direction intercepts the detector,

$$\theta_{CR2} = \arctan\left(\frac{P\cdot\cos\alpha + \sqrt{(RO^2 - P^2\cdot\sin^2\alpha)}}{H}\right)$$

The weighting factor $W1$ associated with this particular selection of θ is

$$W1 = \frac{\int_{\theta_{min}}^{\theta_{max}} \frac{\sin\theta}{2}\cdot d\theta}{\int_0^{\pi} \frac{\sin\theta}{2}\cdot d\theta}$$

and

$$W1 = \frac{\cos\theta_{min} - \cos\theta_{max}}{2}, \quad \text{for } \theta_{max} < \theta_{CR2}$$

$$W1 = \frac{\cos\theta_{min} - \cos\theta_{CR2}}{2}, \quad \text{for } \theta_{max} \geqslant \theta_{CR2}$$

As

$$\alpha \to \alpha_{max} \quad \text{then} \quad W1 \to 0.$$

Results and Discussion

The method was tested by making the geometry approach that of a bare detector and the cylindrical

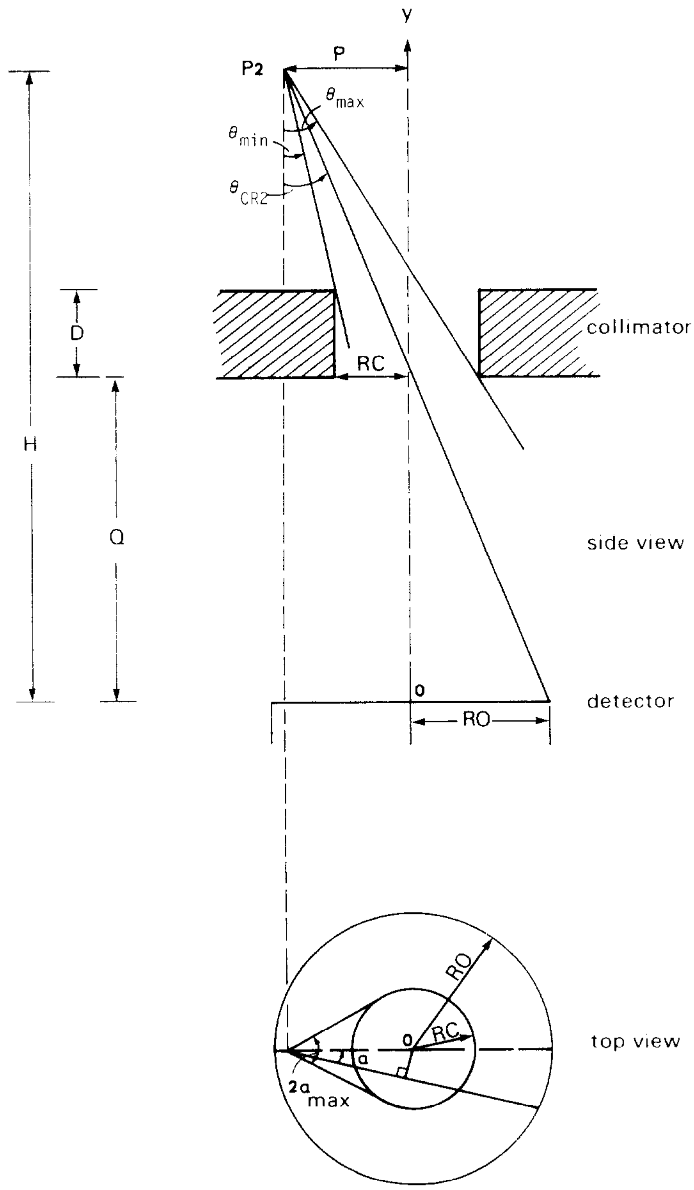

Fig. 3. Geometrical configuration of a point source above the collimator.

target that of a disc shape. For the case when $\epsilon = 0$ (disc target parallel to detector), a comparison between the results obtained and those found in the literature is given in Table 1. The results are in good agreement.

To confirm the results obtained by this method, an experiment was conducted. A uniformly distributed ^{137}Cs disc source of radius 1.35 mm was prepared.

Table 1. Calculated solid angle for a disc source subtended at a bare detector

HO	RC	RS	This work	Gardner (1971)
10	1	1	0.03093	0.03095
10	2	3	0.11475	0.11481
10	3	3	0.25021	0.25045
10	5	6	0.55820	0.55894
10	4	6	0.37091	0.37143
10	5	4.5	0.60044	0.59654

The area under the 662 keV was recorded at two different inclination angles 0° and 90°. The ratio of the peak area at 90° to that at 0° was measured to be 1.14 ± 0.02. The calculated ratio of the average solid angle subtended at the detector by the source at 90° to that at 0° was found to be 1.16 ± 0.02. The two ratios are in good agreement as expected, since in general, the detector response is proportional to the solid angle. This result confirms the validity of the method and emphasizes the importance in considering the orientation of the target when absolute measurements are to be carried out.

In activity measurements, high detection efficiency is achieved by placing the target close to the detector in order to maximise the solid angle subtended between them. However, this results in the solid angle being sensitive to the reproducibility of the experi-

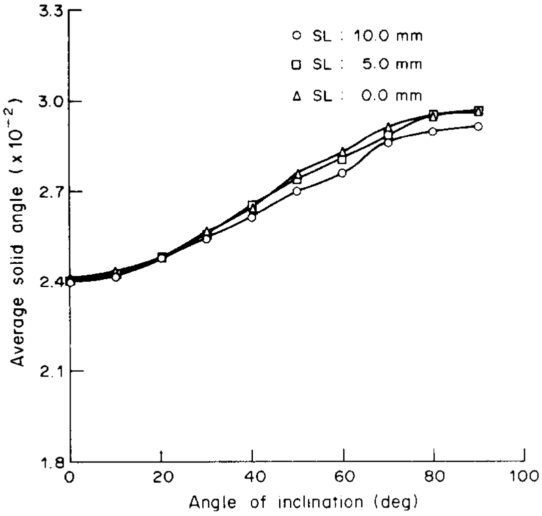

Fig. 4. Variation of solid angle with target rotation for different target thicknesses (*SL*), *in-vitro* case.

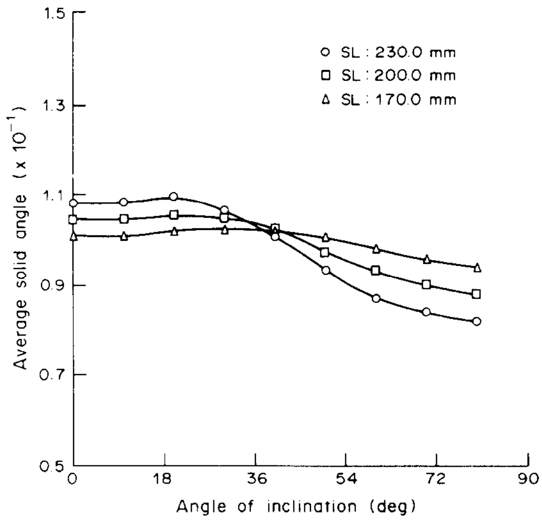

Fig. 6. Variation of solid angle with target rotation for different target thickness (*SL*), *in-vivo* case.

mental set-up. The effect for displacement, along the x-axis, and rotation of the target on the solid angle is investigated for a small target (*in-vitro* case) and a large target (*in-vivo* case) relative to the collimator and target to detector distance.

(*a*) In-vitro *case*

The effect of target rotation on the solid angle, for different target thickness (SL), is shown in Fig. 4. In the regions [0°, 10°] and [80°, 90°] a plateau exists over which the solid angle is independent of rotation. Over the region [0°, 90°] the variation of the solid angle with target thickness 0–10 mm is less than 1.5%. An optimum target position, with respect to rotation, may then be defined at $90° \pm 10°$ over which range the solid angle has a maximum value and is independent of both rotation and target thickness.

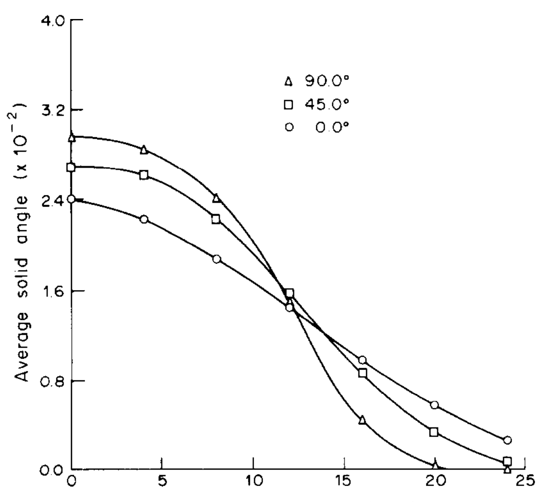

Fig. 5. Variation of solid angle with displacement of a disc target for different angles of inclination, *in-vitro* case.

The effect of displacement of a disc target on the solid angle, for different angles of inclination, is shown in Fig. 5. For small target displacements (< 3 mm), the variation reveals a plateau over which the solid angle is independent of displacement. For displacements greater than 3 mm the variation is rapid, the curves intersect each other and at this point the solid angle is virtually independent of rotation. However, small changes in displacement would introduce large error in the solid angle. Since a target position may be reproduced with a tolerance of less than 3 mm the optimum position at zero displacement is not difficult to achieve.

(*b*) In-vivo *case*

The variation of solid angle with target angle of inclination, for different target thickness, is shown in Fig. 6. In the region [0°, 20°] a plateau exists over which the solid angle is independent of rotation. However, the solid angle is dependent on the target thickness. For an angle of inclination $39° \pm 3°$ the solid angle, for the three cases considered, is the same to within $\pm 1.5\%$ and becomes independent of the target size. This is an important result since in the *in-vivo* measurements the dimensions of the organ being analysed cannot be known very accurately. Also in comparative measurements any differences in size, to within $\pm 15\%$, between the organ and a phantom will have no effect on the final result. The effect of displacement on the solid angle is less than 1% for displacements up to 30 mm.

The method can be easily modified to account for different geometrical configurations and targets of unusual shape. The scheme presented does not include self-absorption and scattering inside the target, although these can be included when generating points within the target (Nakamura, 1970; Horowitz, 1975). Non-isotropic sources, which emit radiation with an angular distribution, may be considered in the analysis by using the appropriate probability density function for this distribution in place of equations (1) and (2). The execution time of the program is directly proportional to the number of sampling histories (N) which in turn controls the error on the estimate of the solid angle (Wielopolski, 1975). For example, for $N = 1000$ and 6000 the execution times on our main frame computer were of the order of 2 and 7s, and the error 3 and 1% respectively.

References

Gardner R. P. and Verghese K. (1971) On the solid angle subtended by a circular disc. *Nucl. Instrum. Methods* **93,** 163.

Horowitz Y. S., Mordechai S. and Dubi A. (1975) Solid-angle and self-absorption factors for an inclined cylindrical source viewed by a cylindrical detector. *Nucl. Instrum. Methods* **123,** 551.

Nakamura T. (1970) Calculation of the detection efficiency of a 3″ × 3″ NaI(Tl) crystal for a thick disc source. *Nucl. Instrum. Methods* **86,** 163.

Nicolaou G. E. (1983) *In-vivo* methods of analysis in the study of elemental composition of the body. PhD. Thesis (Physics), University of Surrey, U.K.

Wielopolski L. (1977) The Monte Carlo calculation of the average solid angle subtended by a right circular cylinder from distributed sources. *Nucl. Instrum. Methods* **143,** 577.